Voronoi Deletion Tilings From Grids

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INTRODUCTION

Question: Which tiling is more natural and mathematically simple to define?

Figure 1: Golconda Fort

Figure 2: Red Fort

In this project, we use Voronoi diagrams to classify specific tilings and tile sets.

BACKGROUND DEFINITIONS

A Voronoi deletion tiling (VDT) is a polygonal tiling of R × R obtained by first deleting finitely many points from its two-dimensional subgrid of points with integer coordinates, and then computing the Voronoi cells of the remaining points.

Definition 1. (Senechal, Definition 2.6, pg 42)**:** Suppose S is a set of points in the plane and let *s* ∈ S be a single point in S. Then the Voronoi cell, V(*s*), is the set of points in the real plane R that lie as least as close to s as to any other point of S.

$$V(s) = \{u \in R^2 \mid |s - u| <= |y - u|, \text{ for all } y \in S^2\}$$

Senechal (pg 43-44) describes how V(s) can be constructed from the point s and its nearest neighbors in S (see Fig. 1):

1. The point *s* is connected to its neighbors in S via line segments.
2. Each such segment is bisected.
3. The Voronoi cell(s) is then the smallest convex region bounded by the bisectors containing s.

Definition 2. Suppose *d* points have been removed from the grid Z^2, and that T is a polygonal tile shape that occurs in the corresponding VDT. Then T occurs at birthday *d*; and if *d* <= *d*' whenever T occurs at birthday *d*', then *d* is the tile birthday of T.

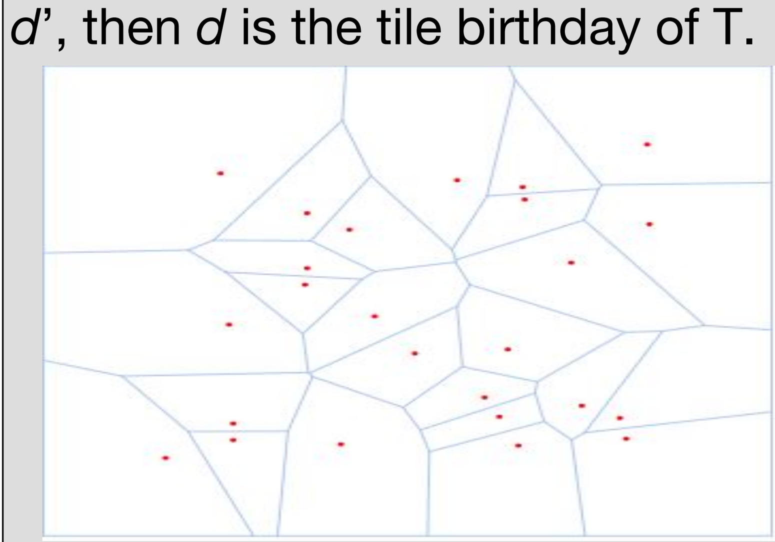


Figure 3: VDT using Mathematica's VoronoiMesh

RESEARCH METHODOLOGY

Our research is pure and experimental. Our data is qualitative, as we will use theorems to explain the distribution of tiles. Specific patterns of coordinates will be deleted from a grid in Mathematica, and we will then classify the resulting shapes.

SIMPLE DELETION PATTERNS

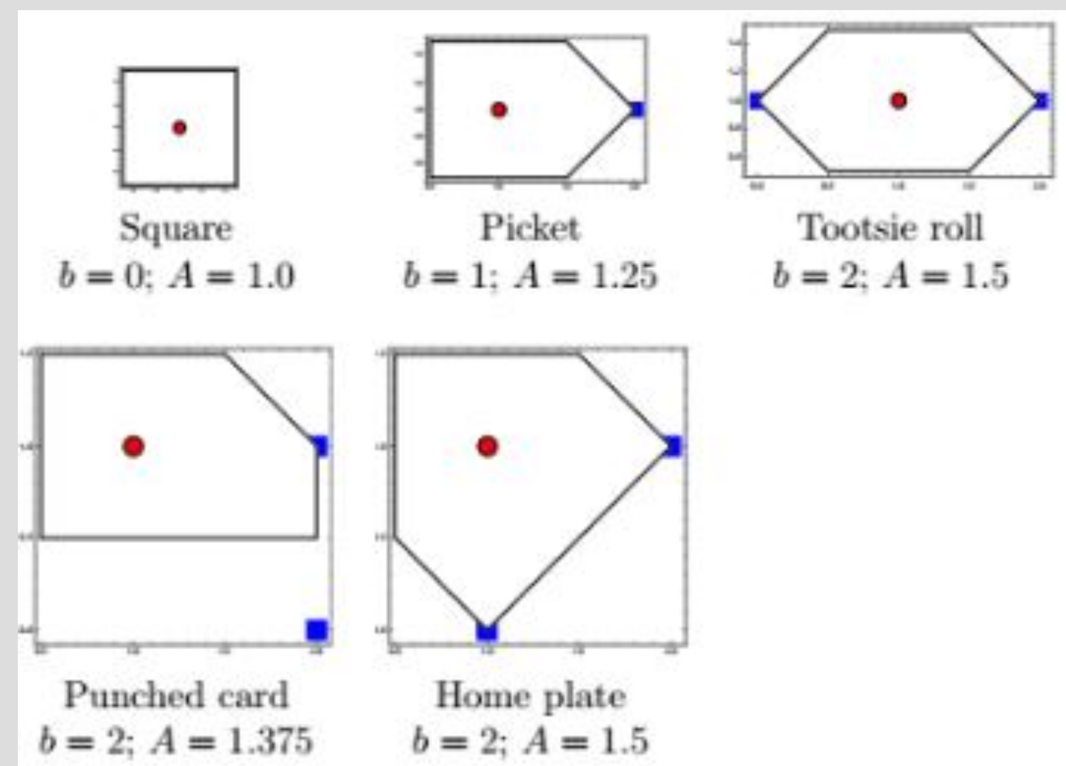


Figure 4: The five Voronoi deletion tiles with birthdays *b* <= 2 and respective areas A.

The associated deleted points of Z^2 are marked by the blue squares; each one is isomorphic to a deletion pattern.

Theorem 1 (Distance 1 Lemma): Suppose D is a finite subset of points that have been deleted from the grid Z^2, and that the Voronoi cells of the remaining points have been computed. Then if *p* is a point not in D that is at distance greater than one from every point in D, then the Voronoi cell V(*p*) is a unit square with center *p*.

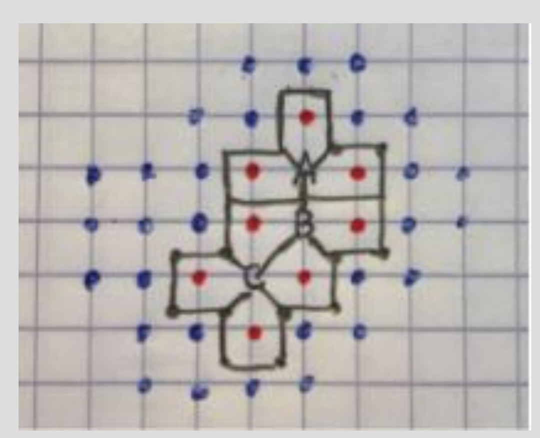


Figure 5: Suppose the three lattice points D = {A, B, C} are deleted from the grid Z^2 as shown.

Proof. By assumption, each of the four points located one unit to the north, east, south and west of *p* is not in D. Therefore the associated Voronoi cell V (*p*) is a unit square with center *p*.

THE CHEAPO KITCHEN TILING

The *cheapo kitchen tiling* arises when a subgrid of points with length 2 distance is deleted. The name was made up due to a lack of other names for it online.

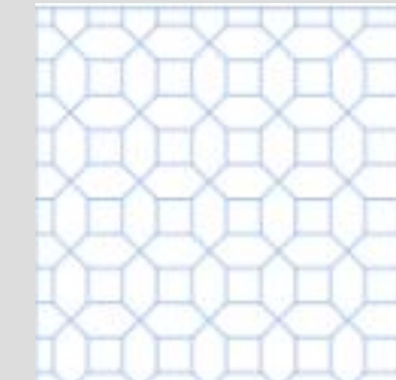
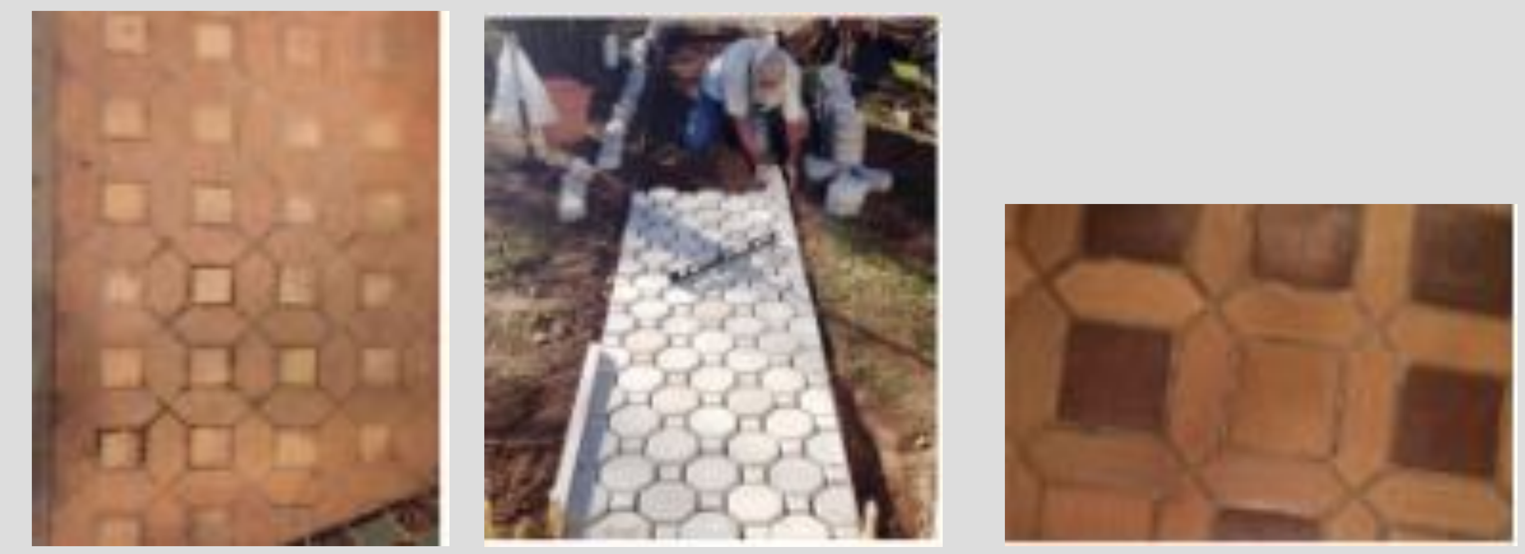


Figure 6 (left): The Cheapo Kitchen Tiling in Mathematica

Figure 7 (below): The Cheapo Kitchen Tiling in real life



The second and third tilings are distorted from the classic by merging different tilings (octagonal. center) or narrowing components (thin tootsie roll, right).

RESULTS

We take a prejudicial view of tilings that do not match an original VDT. We prefer Fig. 1 to Fig 2. because the Red Fort tiling deviates from the cheapo kitchen tiling at the points where it is welded together.

From this work, we can better understand existing architectural characteristics and define new ones.

ACKNOWLEDGEMENTS / REFERENCES

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